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HOMOGENIZATION OF NONLINEAR PARABOLIC BOUNDARY VALUE PROBLEMS IN PERFORATED DOMAINS

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This paper is devoted to the study of the convergence of solutions of boundary value problems for quasilinear parabolic equations in a sequence of perforated domains.

Let Ω be any bounded domain in an n-dimentional Euclidean space R^n and assume that for every natural number s there are a finite number of nonintersecting closed sets $F_i^{(s)}, i = 1, \ldots, I(s)$ contained in Ω. We shall formulate below two types assumptions on $F_i^{(s)}$,from which follow that diameters of $F_i^{(s)}$ tend to zero as $s \to \infty$. By that we have not hypothesis about periodic structure of family $F_1^{(s)}, \ldots, F_{I(s)}^{(s)}$.

We shall consider problem:

$$-\frac{\partial u}{\partial t} + \sum_{j=1}^{n} \frac{d}{dx_j} a_j(x, t, u, \frac{\partial u}{\partial x}) = a_0(x, t, u, \frac{\partial u}{\partial x}), \quad x \in \Omega^{(s)}, t \in [0, T] \qquad (1)$$

$$u(x, t) = f(x, t), \quad x \in \partial\Omega^{(s)}, t \in [0, T] \qquad (2)$$

$$u(x, 0) = g(x), \quad x \in \partial\Omega^{(s)}, \qquad (3)$$

where $\Omega^{(s)} = \Omega \setminus \bigcup_{i=1}^{I(s)} F_i^{(s)}$, $f(x, t), g(x)$ are given functions defined in $\overline{\Omega} \times [0, T]$, $\overline{\Omega}$.

By study these problems the following questions arise: to establish conditions under which the solutions of the problems (1) - (3) converge as $s \to \infty$ and determine the boundary value problem for limit function.

Linear elliptic problems in a sequence of domains with finely granulated boundary have been investigated by V..A. Marchenko and E. Ya. Khruslov (see, e.q. [7]).

The study of this problem in nonlinear case essentially distinguishes from the study of linear problem because by the construction of limit boundary value problem we must have some strong convergence of gradient of solutions of the problems (1) - (3). The proof of such strong convergence is based on special asymptotic expension by which solutions of nonlinear problems in perforated domains are approximated near sets $F_i^{(s)}$ by solutions of model nonlinear problems. Main role by the study of asymptotic behaviour of solutions of nonlinear boundary value problems and by construction of limit boundary value problems have the new type point - wise estimates of solutions of model problems. Corresponding results of author for nonlinear elliptic case there are in papers [1,3,4,10].

In parabolic case the choice of functional space has principal role because by considered conditions the sequence $\frac{\partial u_s}{\partial t}$ can be unbounded in $L_2(\Omega^{(s)} \times [0, T])$, where $u_s(x, t)$ is solution of problem (1) - (3). And we propose the possibility of the study of convergence of sequences $u_s(x, t)$ in space with derivatives of half order with respect to t.

We assume that the functions $a_j(x, t, u, p), j = 0, 1, \ldots, n$ in the equation (1) are defined for all $x \in R^n, t \in R^1, u \in R^1, p \in R^n$ and satisfy the following conditions:

a_1) the functions $a_j(x, t, u, p)$ are continuous in u, p for almost all $(x, t) \in R^n \times R^1$, measurable in x, t for all $(u, p) \in R^1 \times R^n$; $a_j(x, t, 0, 0) \equiv 0$ for $(x, t) \in R^n \times R^1$;

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a_2) there are positive constants ν, μ such that for all values $(x,t) \in R^n \times R^1$, $u, v \in R^1$, $p, q \in R^n$ the inequalities

$$\sum_{j=1}^{n}[a_j(x,u,p) - a_j(x,u,q)](p_j - q_j) \geqslant \nu|p-q|^2,$$
$$|a_j(x,t,u,p) - a_j(x,t,v,q)| \leqslant \mu(|p-q| + |u-v|), j = 1,\dots,n, \quad (4)$$
$$|a_0(x,t,u,p)| \leqslant \mu(|u| + |p|) + \varphi(x,t)$$

hold, where $\varphi(x,t) \in L_2(Q_T)$, $Q_T = \Omega \times [0,T]$.

We suppose for simplicity that $g(x) \equiv 0$ where $g(x)$ is the function from (3). And we assume that the following condition holds:

f) the function $f(x,t)$ in (2) is defined for $x \in \Omega$, $t \in R^1$ is bounded in Q, equal to zero for $t < 0$ and belongs to space $W_2^{1,1/2}(Q)$ where $Q = \Omega \times R^1$.

Let

$$N = \sup\ \mathrm{ess}\{|f(x,t)| : (x,t) \in Q\} + \|f(x,t)\|_{W_2^{1,1/2}(Q)} \quad (5)$$

Used notations for spaces $V_2(Q_T)$, $\overset{o}{V}_2(Q_T)$, $W_2^{1,1/2}(Q)$, $\overset{o}{W}{}_2^{1,1/2}(Q)$ and others are understood so as in [9].

It is easy to prove that for every s the problem (1)-(3) has the solution $u_s(x,t) \in \in V_2(Q_T^{(s)})$, $Q_T^{(s)} = \Omega^{(s)} \times [0,T]$ if T is arbitrary positive number, $g(x) \equiv 0$, conditions a_1), a_2), f) are fulfilled. Moreover, $u_s(x,t)$ belongs to space $W_2^{1,1/2}(Q_T^{(s)})$ and there is a constant M independent of s such that for all s we have the estimates

$$\sup\ \mathrm{ess}\{|u_s(x,t)| : (x,t) \in Q_T^{(s)}\} \leqslant M \quad (6)$$

$$\|u_s(x,t)\|_{V_2(Q_T^{(s)})} \leqslant M, \quad \|u_s(x,t)\|_{W_2^{1,1/2}(Q_T^{(s)})} \leqslant M. \quad (7)$$

We can also prove that for arbitrary function $\psi(x,t) \in \overset{o}{W}{}_2^{1,1/2}(Q_T^{(s)}) \cap W_2^{1,1/2}(Q^{(s)})$ and arbitrary function $\eta(t) \in C^1(R^1)$ with support in interval $(-T,T)$ the integral identity

$$\sqrt{-1}\int_{R^1}\int_{\Omega^{(s)}} \alpha[F(u_s\eta)](x,\alpha)\overline{[F\psi](x,\alpha)}dxd\alpha +$$
$$+\iint_{Q_T^{(s)}}\{u_s(x,t)\psi(x,t)\frac{d\eta(t)}{dt} - \sum_{j=1}^{n} a_j(x,t,u_s,\frac{\partial u_s}{\partial x})\eta(t)\frac{\partial\psi(x,t)}{\partial x_j} - \quad (8)$$
$$-a_0(x,t,u_s,\frac{\partial u_s}{\partial x})\eta(t)\psi(x,t)\}dxdt = 0,$$

is valid and $\eta(t)u_s(x,t) \in W_2^{1,1/2}(Q^{(s)})$. Here $F(u_s\eta)$, $F\psi$ are the Fourier transforms of functions $u_s\eta$, ψ with respect to t, the line over $[F\psi](x,\alpha)$ denote compex conjugation and we put $u_s(x,t) \equiv 0$ for $t < 0$.

The integral identity (8) has principal role by study of the convergence of sequence $\{u_s(x,t)\}$ and by construction of limit boundary value problem.

Denote by $d_i^{(s)}$ the minimum of the radius of the balls containing $F_i^{(s)}$ and let $x_i^{(s)}$ be the center of a ball with radius $d_i^{(s)}$ such that $F_i^{(s)} \subset \overline{B}(x_i^{(s)}, d_i^{(s)})$. Here and in the sequel $B(x_0,\rho)$ denotes the ball with the radius ρ and center at x_0. By $r_i^{(s)}$ we denote the distance of $B(x_i^{(s)}, d_i^{(s)})$ from the set $\bigcup_{j\neq i} B(x_i^{(s)}, d_i^{(s)}) \cup \partial\Omega$.

We suppose that the following conditions are fulfield:

β_1) $d_i^{(s)} \leqslant c_1 r_i^{(s)}$, $\lim_{s\to\infty} r^{(s)} = 0$, $r^{(s)} = \max\{r_1^{(s)},\dots,r_{I(s)}^{(s)}\}$, where c_1 is a constant independent of i, s;

β_2) there is a positive constant c_0 such that the inequality

$$\sum_{i=1}^{I(s)}[d_i^{(s)}]^{2(n-2)}[r_i^{(s)}]^{-n} \leqslant c_0, \quad (9)$$

holds.

Theorem 1. *Let $n > 2$ the conditions $\alpha_1), a_1), f), \beta_1), \beta_2)$ be fulfilled, $g(x) \equiv 0$ and $u_s(x,t)$ is a sequence of solutions of problem (1) - (3) converging weakly to $u_0(x,t)$ in $W_2^{1,1/2}(Q_T)$. Then for every $p \in (1,2), h_0 \in (0,T)$ the following equality*

$$\lim_{s\to\infty} \{supess_{t\in[0,T]} \int_\Omega |u_s(x,t) - u_0(x,t)|^2 dx +$$
$$+ \iint_{Q_T} \left|\frac{\partial u_s(x,t)}{\partial x} - \frac{\partial u_0(x,t)}{\partial x}\right|^p dxdt + \tag{10}$$
$$+ \sup_{0<h<h_0} ess \iint_{Q_{T-h_0}} \left|\frac{u_s(x,t+h) - u_0(x,t+h) - u_s(x,t) + u_0(x,t)}{\sqrt{h}}\right|^p dxdt\} = 0$$

holds.

To construct a limit boundary value problem we need in additional assumption.

We denote $\lambda_s = [\ln 1/r^{(s)}]^{-1}, s = 1,2,\ldots$ and define the numerical sequence $\rho_i^{(s)}$ by equalities

$$\rho_i^{(s)} = 2d_i^{(s)} \quad \text{if} \quad i \in I'(s) = \{i = 1,\ldots,I(s) : d_i^{(s)} \geqslant [r_i^{(s)}]^{\frac{n}{n-2}} \lambda_s^{-1}\},$$
$$\rho_i^{(s)} = [r_i^{(s)}]^{\frac{n}{n-2}} \lambda_s^{-2} \quad \text{if} \quad i \in I''(s) = \{i = 1,\ldots,I(s) : d_i^{(s)} < [r_i^{(s)}]^{\frac{n}{n-2}} \lambda_s^{-1}\} \tag{11}$$

We can suppose that s is so great that the inequalities $\rho_i^{(s)} \leqslant \frac{1}{4} r_i^{(s)}$ for $i \in I''(s), d_i^{(s)} \leqslant$
$\leqslant \frac{1}{2}, \lambda_s < \frac{1}{16}$ hold.

For $s = 1,2,\ldots$ and $i \in I''(s)$ we divide the segment $[0,T]$ on $K(i,s)$ segments of equal length by points $t_{i,0}^{(s)} = 0, t_{i,1}^{(s)},\ldots,t_{i,K(i,s)}^{(s)} = T$ so that the inequalities $\frac{1}{2}[\rho_i^{(s)}]^2 \leqslant$
$\leqslant t_{i,k}^{(s)} - t_{i,k-1}^{(s)} \leqslant [\rho_i^{(s)}]^2$ hold.

We define for $i \in I''(s), s = 1,2,\ldots$ and $q \in R^1$ a function $v_i^{(s)}(x,t,q)$ as a solution of the next boundary value problem

$$\frac{\partial v}{\partial t} - \sum_{j=1}^n \frac{d}{dx_j} a_j(x,t,0,\frac{\partial v}{\partial x}) = 0 \quad \text{for} \quad (x,t) \in G_i^{(s)} \times [-T,T+1],$$
$$v(x,t) = q\omega(|x - x_i^{(s)}|)\omega(-\frac{t}{T}) \quad \text{for} \quad (x,t) \in \partial G_i^{(s)} \times [-T,T+1], \tag{12}$$
$$v(x,-T) = 0$$

where $G_i^{(s)} = B(x_i^{(s)},1) \setminus F_i^{(s)}$, $\omega(r)$ is a fixed function of class $C^\infty(R^1)$, equal to one for $r \leqslant \frac{1}{2}$, to zero for $r \geqslant 1$ and such that $0 \leqslant \omega(r) \leqslant 1$.

We formulate an additional condition

h) There exists a continuons function $c(x,t,q)$ such that for an arbitrary ball $B \subset Q_T$ the equality

$$\lim_{s\to\infty} \sum_{(i,k)\in I_s(B)} \sum_{j=1}^n \frac{1}{q} \iint_{Q_{ik}^{(s)}} a_j(x,t,0,\frac{\partial v_{ik}^{(s)}(x,t,q)}{\partial x}) \frac{\partial v_{ik}^{(s)}(x,t,q)}{\partial x_j}) dxdt =$$
$$= \iint_B c(x,t,q) dxdt, \quad Q_{ik}^{(s)} = B(x_i^{(s)}, 2\rho_i^{(s)}) \times [t_{i,k-1}^{(s)}, t_{i,k}^{(s)}] \tag{13}$$

holds and the limit in (13) is uniform with respect to q on every bounded interval. In (13) $I_s(B)$ is the set of those pairs (i,k) for which $i \in I''(s), k = 1,\ldots,K(i,s)$ and $(x_i^{(s)}, t_{i,k}^{(s)}) \in B$.

Theorem 2. *Let the conditions of theorem 1 and condition b be fulfilled. Then the function $u_0(x,t)$ is a solution of the problem*

$$\frac{\partial u}{\partial t}-\sum_{j=1}^{n}\frac{d}{dx_j}a_j(x,t,u,\frac{\partial u}{\partial x})=c(x,t,f-u)-a_0(x,t,u,\frac{\partial u}{\partial x}),\quad (x,t)\in Q_T,$$
$$u(x,t)=f(x,t),\quad x\in\partial\Omega,\quad t\in[0,T], \tag{14}$$
$$u(x,0)=0,\quad x\in\Omega.$$

Proofs of the Theorems 1,2 are based on special a priori estimates of solutions which were proved by the author in [5].

Theorem 3. *Let $n>2$ the conditions $a_1),a_2)$ be fulfilled. There is a constant K_3 depending only on ν,μ,n,T such that for the solution of problem (12) the inequality*

$$|v_i^{(s)}(x,t,q)|\leqslant|q|\min\left\{K_3\left(\frac{d_i^{(s)}}{|x-x_i^{(s)}|}\right)^{n-2},1\right\} \tag{15}$$

is valid for $(x,t)\in G_i^{(s)}\times[-T,T]$.

It is possible to formulate the inequality (15), in terms of capacities analogously to the estimates in elliptic case [3,4].

Construction of the asymptotic expansion is connected with separation the leading terms which are constructed by solutions of local boundary value problems.

Corresponding to subdivision of segment $[0,T]$ by points $t_{i,k}^{(s)}$, $k=0,1,\ldots,K(i,s)$ we define the infinitely differentiable functions $g_{ik}^{(s)}(t),\bar{g}_{il}^{(s)}(t)$ for $t\in R^1, s=1,2,\ldots,k=1,\ldots,K(i,s), l=0,1,\ldots,K(i,s), i\in I''(s)$ satisfying the conditions:

1) the supports of functions $g_{ik}^{(s)}(t),\bar{g}_{il}^{(s)}(t)$ are contained correspondingly in intervals $(t_{i,k-1}^{(s)}+\lambda_s[\rho_i^{(s)}]^2,t_{i,k}^{(s)}-\lambda_s[\rho_i^{(s)}]^2),(t_{i,l}^{(s)}-2\lambda_s[\rho_i^{(s)}]^2,t_{i,l}^{(s)}+2\lambda_s[\rho_i^{(s)}]^2)$; the values of these functions belong to segment $[0,1]$;

2) for $t\in[0,T]$ the identity

$$\sum_{i=1}^{K(i,s)}g_{ik}^{(s)}(t)+\sum_{l=0}^{K(i,s)}\bar{g}_{il}^{(s)}(t)\equiv 1 \tag{16}$$

holds;

3) for all values of $t\in R^1, s=1,2,\ldots,i\in I''(s)$ the inequalities

$$\left|\frac{dg_{ik}^{(s)}(t)}{dt}\right|\leqslant 2\lambda_s^{-1}[\rho_i^{(s)}]^{-2},\qquad \left|\frac{d\bar{g}_{il}^{(s)}(t)}{dt}\right|\leqslant 2\lambda_s^{-1}[\rho_i^{(s)}]^{-2}$$

are valid.

For $s=1,2,\ldots$ and $i\in I'(s)$ we divide the segment $[0,T]$ on $R(i,s)$ segments of equal length by points $\tilde{t}_{i,r}^{(s)}, r=0,1,\ldots,R(i,s)$ such that $\tilde{t}_{i,0}^{(s)}=0,\tilde{t}_{i,R(i,s)}^{(s)}=T$ and $\frac{1}{2}[d_i^{(s)}]^2R(i,s)\leqslant T\leqslant[d_i^{(s)}]^2R(i,s)$.

We define for $t\in R^1, s=1,2,\ldots,i\in I'(s), r=0,1,\ldots,R(i,s)$ the infinitely differentiable functions $\tilde{g}_{ir}^{(s)}(t)$ satisfying the conditions:

1) the support of function $\tilde{g}_{ir}^{(s)}(t)$ is contained in interval $(\tilde{t}_{i,r}^{(s)}-[d_i^{(s)}]^2,\tilde{t}_{i,r}^{(s)}+[d_i^{(s)}]^2)$; the values of this function belong to segment $[0,1]$;

2) for $t\in[0,T]$ the identity

$$\sum_{r=0}^{R(i,s)}\tilde{g}_{ir}^{(s)}(t)\equiv 1 \tag{17}$$

holds;

3) for all values of $t \in R^1, s = 1, 2, \dots, i \in I'(s)$ the inequalities

$$\left|\frac{d\tilde{g}_{ir}^{(s)}}{dt}\right| \leqslant 2[d_i^{(s)}]^{-2}$$

are valid.

We define also the functions $\varphi_i^{(s)}(x), i = 1, \dots, I(s), \psi_i^{(s)}(x), i \in I''(s), s = 1, 2, \dots$ by equalities

$$\varphi_i^{(s)}(x) = \omega\left(\frac{|x - x_i^{(s)}|}{2\rho_i^{(s)}}\right), \quad \psi_i^{(s)}(x) = \omega\left(\frac{|x - x_i^{(s)}|}{2\sqrt{\lambda_s}\rho_i^{(s)}}\right)$$

where $\omega(r)$ is the same function as in (12).

We denote by $Q_{i,k}^{(s)}, \overline{Q}_{i,l}^{(s)}, \tilde{Q}_{i,r}^{(s)}$ correspondingly cylinders

$$B(x_i^{(s)}, 2\rho_i^{(s)}) \times [t_{i,k-1}^{(s)}, t_{i,k}^{(s)}], B(x_i^{(s)}, 2\sqrt{\lambda_s}\rho_i^{(s)}) \times$$
$$\times [t_{i,l}^{(s)} - 2\lambda_s(\rho_i^{(s)})^2, t_{i,l}^{(s)} + 2\lambda_s(\rho_i^{(s)})^2],$$
$$B(x_i^{(s)}, 2d_i^{(s)}) \times [\tilde{t}_{i,r}^{(s)} - 2(d_i^{(s)})^2, \tilde{t}_{i,r}^{(s)} + 2(d_i^{(s)})^2].$$

And let for arbitrary cylinder Q' and integrable function $g(x,t)$

$$M[g, Q'] = \frac{1}{\text{mes } Q'} \iint\limits_{Q'} g(x,t)\,dx\,dt$$

be the mean value of $g(x,t)$ with respect to Q'.

We define

$$u_{ik}^{(s)} = M[u_0, Q_{ik}^{(s)}], \overline{u}_{i,l}^{(s)} = M[u_0, \overline{Q}_{i,l}^{(s)}], \tilde{u}_{i,r}^{(s)} = M[u_0, \tilde{Q}_{i,r}^{(s)}],$$
$$f_{ik}^{(s)} = M[f, Q_{ik}^{(s)}], \overline{f}_{i,l}^{(s)} = M[f, \overline{Q}_{i,l}^{(s)}], \tilde{f}_{i,r}^{(s)} = M[f, \tilde{Q}_{i,r}^{(s)}]$$

where $u_0(x,t)$ is weak limit of sequence $u_s(x,t)$, $f(x,t)$ is a function from boundary condition (2).

We define the asymptotic expansion by equality

$$u_s(x,t) = u_0(x,t) + r_s(x,t) + \sum_{j=1}^{5} r_s^{(j)}(x,t) + w_s(x,t) \tag{18}$$

where

$$\begin{aligned}
r_s(x,t) &= \sum_{i \in I''(s)} \sum_{k=1}^{K(i,s)} v_i^{(s)}(x,t,f_{i,k}^{(s)} - u_{i,k}^{(s)}) g_{i,k}^{(s)}(t) \varphi_i^{(s)}(x), \\
r_s^{(1)}(x,t) &= \sum_{i \in I'(s)} \sum_{r=0}^{R(i,s)} v_i^{(s)}(x,t,\tilde{f}_{i,r}^{(s)} - \tilde{u}_{i,r}^{(s)}) \tilde{g}_{ir}^{(s)}(t) \varphi_i^{(s)}(x), \\
r_s^{(2)}(x,t) &= \sum_{i \in I''(s)} \sum_{l=0}^{K(i,s)} v_i^{(s)}(x,t,\overline{f}_{i,l}^{(s)} - \overline{u}_{i,l}^{(s)}) \overline{g}_{i,l}^{(s)}(t) \psi_i^{(s)}(x), \\
r_s^{(3)}(x,t) &= \sum_{i \in I'(s)} \sum_{r=0}^{R(i,s)} \{[\tilde{u}_{i,r}^{(s)} - u_0(x,t)] + [f(x,t) - \tilde{f}_{i,r}^{(s)}]\} \tilde{g}_{i,r}^{(s)}(t) \varphi_i^{(s)}(x), \\
r_s^{(4)}(x,t) &= \sum_{i \in I''(s)} \sum_{k=1}^{K(i,s)} \{[u_{i,k}^{(s)} - u_0(x,t)] + [f(x,t) - f_{i,k}^{(s)}]\} g_{i,k}^{(s)}(t) \varphi_i^{(s)}(x), \\
r_s^{(5)}(x,t) &= \sum_{i \in I''(s)} \sum_{l=0}^{K(i,s)} \{[\overline{u}_{i,l}^{(s)} - u_0(x,t)] + [f(x,t) - \overline{f}_{i,l}^{(s)}]\} \overline{g}_{i,l}^{(s)}(t) \psi_i^{(s)}(x),
\end{aligned} \tag{19}$$

and $w_s(x,t)$ is a remainder term of expansion and this function is defined by equality (18) for $x \in \Omega$, $t \in (-\infty, T)$ if we put $v_i^{(s)}(x,t,q) \equiv 0$ for $x \in \Omega$, $t \notin [-T, T+1]$.

By study of behaviour of $r_s(x,t)$ and $r_s^{(j)}(x,t)$ we use the pointwise estimate (15) and integral estimates for $\frac{\partial v_i^{(s)}(x,t,q)}{\partial x}$ which it is possible prove using the estimate (15). In such way we can prove the next results.

Theorem 4. *Let $n > 2$ the conditions $a_1), a_2), \beta_1), \beta_2), f)$ be fulfilled. Then the sequences $r_s^{(j)}(x,t)$, $j = 1, 2, 3, 4, 5$ converge strongly to zero in spaces $V_2(Q_T)$, $W_2^{1,\frac{1}{2}}(Q)$.*

Theorem 5. *Let $n > 2$ the conditions $a_1), a_2), \beta_1), \beta_2), f)$ be fulfilled. Then the sequence $r_s(x,t)$ is bounded in spaces $V_2(Q_T)$, $W_2^{1,\frac{1}{2}}(Q)$ and for arbitrary $p \in (1,2)$ the equality*

$$\lim_{s\to\infty} \{ sup\ ess_{t \in R^1} \int_\Omega |r_s(x,t)|^2 dx + \iint_Q \left| \frac{\partial r_s(x,t)}{\partial x} \right|^p dxdt + \\ + sup\ ess_{h>0} \iint_Q \left| \frac{r_s(x,t+h) - r_s(x,t)}{\sqrt{h}} \right|^p dxdt \} = 0 \tag{20}$$

holds.

Using the theorems 4, 5 we can study the behaviour of the remainder term of asymptotic expansion (18). The next theorem is valid.

Theorem 6. *Let $n > 2$ the conditions $a_1), a_2), \beta_1), \beta_2, f)$ be fulfilled, $g(x) \equiv 0$, $u_s(x,t)$ is the solution of problem (1)-(3) and let the sequence $u_s(x,t)$ converge weakly to $u_0(x,t)$ in $W_2^{1,\frac{1}{2}}(Q_T)$. Then the equality*

$$lim_{s\to\infty} \|w_s(x,t)\eta(t)\|_{W_2^{1,\frac{1}{2}}(Q)} = 0 \tag{21}$$

holds for arbitrary function $\eta(t) \in C^1(R^1)$ with support in interval $(-T,T)$.

For proof of last theorem we insert in integral identity (8) the function $w_s(x,t)\eta(t)$ instead of test function $\psi(x,t)$.
From theorems 4, 6 we obtain that

$$lim_{s\to\infty} \|u_s(x,t) - u_0(x,t) - r_s(x,t)\|_{W_2^{1,\frac{1}{2}}(Q_{T'})} = 0$$

for $T' < T$ so we can consider $r_s(x,t)$ as a corrector with respect to convergence in $W_2^{1,\frac{1}{2}}$.
Now we shall demonstrate the method of the derivation of the boundary value problems for the limit function u_0.
Let $u_m^{(0)}(x,t)$ be a sequence of functions from $C^\infty(Q)$ such that

$$u_m^{(0)}(x,t) \equiv 0 \quad \text{for} \quad t < 0, \quad |u_m^{(0)}(x,t)| \leqslant M \quad \text{for} \quad (x,t) \in Q, \\ \|u_m^{(0)}(x,t) - u_0(x,t)\|_{W_2^{1,\frac{1}{2}}(Q_T)} \to 0, \quad \text{if} \quad m \to \infty. \tag{22}$$

Let $h(x,t)$ be an arbitrary function of class $C_0^\infty(Q)$ and for $m = 1, 2, \ldots$ let us introduce sequences

$$h_{sm}(x,t) = h(x,t) - \sum_{j=1}^{2} \rho_{sm}^{(j)}(x,t) - \sum_{j=1}^{3} \rho_s^{(j)}(x,t), \tag{23}$$

where

$$\rho_{sm}^{(1)}(x,t) = \sum_{(i,k)\in J_{sm}^{(1)}} v_i^{(s)}(x,t,f_{ik}^{(s)} - u_{ikm}^{(s)}) \frac{h_{ik}^{(s)}}{f_{ik}^{(s)} - u_{ikm}^{(s)}} g_{ik}^{(s)}(t)\varphi_i^{(s)}(x),$$

$$\rho_{sm}^{(2)}(x,t) = \sum_{(i,k)\in J_{sm}^{(2)}} v_i^{(s)}(x,t,1) h_{ik}^{(s)} g_{ik}^{(s)}(t)\varphi_i^{(s)}(x),$$

$$\rho_s^{(3)}(x,t) = \sum_{i\in I'(s)} \sum_{r=0}^{R(i,s)} \{[v_i^{(s)}(x,t,1) - 1]\tilde{h}_{ir}^{(s)} + h(x,t)\}\tilde{g}_{ir}^{(s)}(t)\varphi_i^{(s)}(x), \tag{24}$$

$$\rho_s^{(4)}(x,t) = \sum_{i\in I''(s)} \sum_{l=0}^{K(i,s)} \{[v_i^{(s)}(x,t,1) - 1]\overline{h}_{il}^{(s)} + h(x,t)\}\overline{g}_{il}^{(s)}(t)\psi_i^{(s)}(x),$$

$$\rho_s^{(5)}(x,t) = \sum_{i\in I''(s)} \sum_{k=1}^{K(i,s)} [h(x,t) - h_{ik}^{(s)}] g_{ik}^{(s)}(t)\varphi_i^{(s)}(x)$$

Here we keep all preceeding notations and

$$u_{ikm}^{(s)} = M[u_m^{(0)}; Q_{ik}^{(s)}], \quad h_{ik}^{(s)} = M[h, Q_{ik}^{(s)}],$$
$$\overline{h}_{ir}^{(s)} = M[h, \overline{Q}_{ir}^{(s)}], \quad \tilde{h}_{il}^{(s)} = M[h, \tilde{Q}_{il}^{(s)}],$$
$$J_{sm}^{(1)} = \{(i,k) : i \in I''(s), k = 1,\dots,K(i,s), \ |f_{ik}^{(s)} - u_{ikm}^{(s)}| \geqslant d_i^{(s)}\},$$
$$J_{sm}^{(2)} = \{(i,k) : i \in I''(s), k = 1,\dots,K(i,s), \ |f_{ik}^{(s)} - u_{ikm}^{(s)}| < d_i^{(s)}\}.$$

Using the pointwise estimate (15) it is possible to prove next theorem.

Theorem 7. *Let $n > 2$ the conditions a_1, a_2, β_1), β_2) be fulfilled. There are exist a positive constant K non depending on s, m and sequence γ_s tending to zero with $s \to \infty$ such that the inequalities*

$$\iint_Q \left|\frac{\partial\rho_{sm}^{(1)}(x,t)}{\partial x}\right|^2 dxdt +$$
$$+ \ supess_{\tau>0} \iint_Q \frac{|\rho_{sm}^{(1)}(x,t+\tau) - \rho_{sm}^{(1)}(x,t)|^2}{\tau} dxdt \leqslant K\|h\|_{C^0(Q)},$$

$$supess_{t\in R^1} \int_\Omega |\rho_{sm}^{(1)}(x,t)|^2 dx + \iint_Q \left|\frac{\partial\rho_{sm}^{(1)}(x,t)}{\partial x}\right|^p dxdt + \tag{25}$$
$$+ \ supess_{\tau>0} \iint_Q \left|\frac{\rho_{sm}^{(1)}(x,t+\tau) - \rho_{sm}^{(1)}(x,t)}{\sqrt{\tau}}\right|^p dxdt \leqslant \gamma_s\|h\|_{C^0(Q)},$$

$$\|\rho_{sm}^{(2)}(x,t)\|_{W_2^{1,\frac{1}{2}}(Q)} + \sum_{j=3}^{5} \|\rho_s^{(j)}(x,t)\|_{W_2^{1,\frac{1}{2}}(Q)} \leqslant \gamma_s\|h\|_{C^0(Q)}$$

hold where p is arbitrary number from interval $(1,2)$.

From (23), (24), properties of $v_i^{(s)}(x,t,q)$, $g_{ik}^{(s)}(t)$, $\varphi_i^{(s)}(x)$ and other functions it is followed that

$$h_{sm}(x,t) \in \overset{o}{W}{}_2^{1,\frac{1}{2}}(Q_T^{(s)}) \cap W_2^{1,\frac{1}{2}}(Q^{(s)}), \ s,m = 1,2,\dots \tag{26}$$

So we can insert $h_{sm}(x,t)$ in integral identity (8) instead of test function $\psi(x,t)$.

After such substitution and evaluation of arising terms we can prove that the equality

$$
\begin{aligned}
&\sqrt{-1}\int_{R^1}\int_\Omega \alpha[F(u_0\eta)](x,\alpha)\overline{[Fh](x,\alpha)}dxd\alpha+\\
&+\iint_{Q_T}\{u_0(x,t)h(x,t)\frac{d\eta(t)}{dt}-\sum_{j=1}^n a_j(x,t,u_0,\frac{\partial u_0}{\partial x})\eta(t)\frac{\partial h(x,t)}{\partial x_j}-\\
&-a_0(x,t,u_0,\frac{\partial u_0}{\partial x})\eta(t)h(x,t)\}dxdt+ \qquad (27)\\
&+\sum_{i=1}^{I(s)}\sum_{k=1}^{K(i,s)}\frac{h_{ik}^{(s)}}{f_{ik}^{(s)}-u_{ikm}^{(s)}}\sum_{j=1}^n\int_{t_{i,k-1}^{(s)}}^{t_{i,k}^{(s)}}\int_\Omega a_j(x,t,0,\frac{\partial v_i^{(s)}(x,t,f_{ik}^{(s)}-u_{ikm}^{(s)})}{\partial x})\times\\
&\times\frac{\partial v_i^{(s)}(x,t,f_{ik}^{(s)}-u_{ikm}^{(s)})}{\partial x_j}\eta(t)dxdt=R(m,s)
\end{aligned}
$$

holds where for $R(m,s)$ the inequality

$$|R(m,s)|\leqslant\gamma_s^{(1)}+\gamma_m^{(2)} \qquad (28)$$

is valid and $\gamma_s^{(1)},\gamma_m^{(2)}$ tend to zero if $s,m\to\infty$. By proof of (27) we use also result of theorem 1 about strong convergence of $\frac{\partial u_s(x,t)}{\partial x}$ in $L_p(Q_T)$.

Further we can prove that last summand of left-hand side of equality (27) tends to

$$\iint_{Q_T} c(x,t,f(x,t)-u_m^{(0)}(x,t))h(x,t)\eta(t)dxdt \qquad (29)$$

if $s\to\infty$ on the basis of the condition h. And passing to limit in (27) if $s\to\infty$ and further if $m\to\infty$ we receive that the function $u_0(x,t)$ is the solution of boundary value problem (14).

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